Subject : 1.1 linear algebra

Mode :

Matrix :

$A_{m\times n} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{bmatrix} 3 & 7 & -2 \\ 6 & 1 & 0 \\ 10 & 8 & 5 \end{bmatrix}$ m: number of rows, n: number of Columns

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \end{pmatrix}$$

a_{ij} is element at intersection row i and Column j

- if $m=n$ ∴ it's square matrix
  number of rows = number of Columns

- $A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$ a_{ii} is the diagonal.

- Trace of square matrix A is given by $Tr(A) = \sum_{i=1}^{n} a_{jj}$, $m=n$
  ∴ is summation of elements of the diagonal
- $A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$ $Tr(A) = 1+2 = 3$
- if $Tr(A) = 0 \Rightarrow$ ∴ it's A Traceless matrix

Sum & Sub of Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$A+B = B+A$

- $A \pm B = C = \begin{pmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} & a_{13} \pm b_{13} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} & a_{23} \pm b_{23} \\ a_{31} \pm b_{31} & a_{32} \pm b_{32} & a_{33} \pm b_{33} \end{pmatrix}$

Matrices multiplication :

$A = \begin{pmatrix} 3 & 5 & 1 \\ 4 & 6 & 7 \end{pmatrix}$ k is number, scale ±, Complex

$A \cdot k = \begin{pmatrix} 3k & 5k & k \\ 4k & 6k & 7k \end{pmatrix}$

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Date : Subject :

- $A_{m\times n} \cdot B_{n\times k} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ a_{31} & a_{32} & \cdots & a_{3m} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & & & \vdots \\ \vdots & & & \\ b_{m1} & & & b_{mk} \end{pmatrix}$

number of Columns in ~~Mat~~ First Matrix = number of rows in Second matrix $\Longrightarrow$ To multiply Two Matrices

$AB \neq BA$ Generally

- Transpose of A :-

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $A^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$

$a_{ij} \xrightarrow{\text{Transpose}} a_{ji}$, $(AB)^T = B^T A^T$, $(ABCD)^T = D^T C^T B^T A^T$

- Identity matrix :- $I_A = AI = A$

$I_{nn} = \begin{pmatrix} 1 & 0 & 0 & \cdots & \\ 0 & 1 & 0 & \cdots & \\ 0 & 0 & 0 & \cdots & \\ 0 & 0 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$ $a_{ij} = 0, i \neq j$

- $C = AB$

$(AB)_{ij} = C_{ij} = \sum_{m=1}^{n} A_{im} B_{mj}$

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$

- EX: $A = \begin{pmatrix} 1 \rightarrow 2 \\ 3 \quad 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

$C = AB = \begin{pmatrix} 1\times5 + 2\times7 & 1\times6 + 2\times8 \\ 3\times5 + 4\times7 & 3\times6 + 4\times8 \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$

Inverse :- $AA^{-1} = A^{-1}A = I$

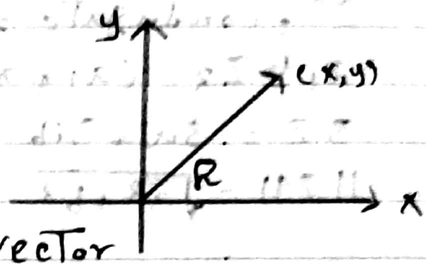
Subject : Mode :

– Vectors :-

$A = [a \ b \ c \ d \ e]$ row vector , $B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix}$ Column Vector

* Geometric

$R = \begin{bmatrix} x \\ y \end{bmatrix}$



$R = \sqrt{x^2 + y^2}$

$R^T R = \|R\| = |R| =$ Norm

length of the row vector and Column vector

• ex :- $R = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $R^T = [1 \ 2]$

$R^T R = 1^2 + 2^2 = 5$

$\|R\| = |R| = \sqrt{R^T R} = \sqrt{5}$

– Orthognal vector :- A, B row vectors

$A \cdot B = B \cdot A^T = AB^T = 0$

$A = [1 \ 0 \ 0 \ 0]$, $B = [0 \ 0 \ 0 \ 1]$

$A \cdot B = AB^T = [1 \ 0 \ 0 \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$

– Orthognal matrix ⇒ graphics (rotation)

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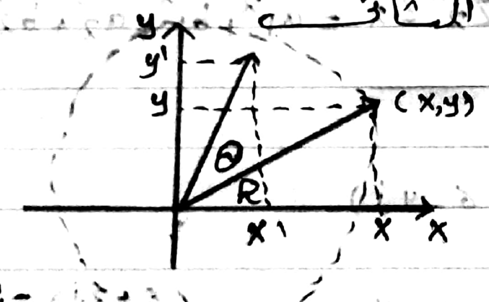

ex :-

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$V_1 = [\cos\theta \ \ -\sin\theta]$, $V_2 = [\sin\theta \ \ \cos\theta]$

$V_1 V_2^T = [\cos\theta \ \ -\sin\theta] \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} = 0$

Date : Subject :

- Complex Number

$Z = a + ib$ a, b numbers

Z^* : Condugate $= a - ib$ تغيير اشارة العدد التخيلي فقط

$Z_1 \pm Z_2 = (a_1 + a_2) + i(b_1 \pm b_2)$

$3Z = 3a + 3ib$

$\|Z\| = \sqrt{a^2 + b^2} = Z.Z^*$

- Complex Matrix

$A = \begin{pmatrix} 3+5i & 6 \\ 8j & 7-2j \end{pmatrix}$

$A^* = \begin{pmatrix} 3-5i & 6 \\ -8j & 7+2j \end{pmatrix}$

- $A = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ Z_1, Z_2 Complex

$A^T A = (Z_1 \; Z_2)\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = Z_1^2 + Z_2^2$ # Not real number

$- Z = 3 + 2j \Rightarrow Z^2 = 5 + 12j$ Not real

- $A = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$

$A^{*T} A = (Z_1^* \; Z_2^*)\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = Z_1^* Z_1 + Z_2^* Z_2 = a_1^2 + b_1^2 + a_2^2 + b_2^2$

- ex: $Z = 3 + 2j$, $ZZ = Z^2 = 5 + 4j$

$A = \begin{pmatrix} 5+2j \\ 6-4j \end{pmatrix}$ $A^T = (5+2j \quad 6-4j)$

$A^{*T} = (5-2j \quad 6+4j)$, $A^{*T} A = (5-2j \quad 6+4j)\begin{pmatrix} 5+2j \\ 6-4j \end{pmatrix}$

$= 25 + 4 + 36 + 16 = 81$

$- j^2 = -1$

- Normalization : $|A| \neq 1 \longrightarrow \|\bar{A}\| = 1$

Subject :   Mode :

EX: $A = [1 \quad 2]$

$AA^T = [1 \quad 2]\begin{bmatrix}1\\2\end{bmatrix} = 1+4 = 5 \neq 1$

$\bar{A} = \frac{1}{\sqrt{\|A\|}} A = \frac{1}{\sqrt{5}} [1 \quad 2] = [\frac{1}{\sqrt{5}} \quad \frac{2}{\sqrt{5}}]$

$\|\bar{A}\| = \sqrt{(1/5)+(4/5)} = 1$

- Common Factor:-

$A = \begin{pmatrix}4 & 3\\2 & 1\end{pmatrix}$

$A = 3\begin{pmatrix}4/3 & 1\\2/3 & 1/3\end{pmatrix}$

- $A = \begin{pmatrix}1 & 3\\4 & 2\end{pmatrix}$ $Tr(A) = 1+2 = 3$

$A^T = \begin{pmatrix}1 & 4\\3 & 2\end{pmatrix}$ $Tr(A^T) = 1+2 = 3$

$Tr(A) = Tr(A^T)$

- Special Vectors:-

$u_1 = \begin{pmatrix}1\\0\end{pmatrix}, u_2 = \begin{pmatrix}0\\1\end{pmatrix}$

$u_1^T u_2 = (1 \quad 0)\begin{pmatrix}0\\1\end{pmatrix} = 0 \quad \therefore u_1 \perp u_2$

$\|u_1\| = \sqrt{u_1^T u_1} = \sqrt{(1 \quad 0)\begin{pmatrix}1\\0\end{pmatrix}} = 1$ (unit vector)

$\|u_2\| = \sqrt{u_2^T u_2} = \sqrt{(0 \quad 1)\begin{pmatrix}0\\1\end{pmatrix}} = 1$ (unit vector)

- $A = \begin{pmatrix}3\\5\end{pmatrix} = 3\begin{pmatrix}1\\0\end{pmatrix} + 5\begin{pmatrix}0\\1\end{pmatrix}$
- $X = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

$Xu_1 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = u_2$ • we can consider that u_1 = on (1)

$Xu_2 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} = u_1$ and u_2 = off (0)

- $H = \begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$, $Hu_1 = \begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}$

$Hu_2 = \begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}$