lec one matrix

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ or $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A_{m \times n}$

$m \Rightarrow$ no. of rows

$n \Rightarrow$ no. of columns

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}$$

* element a_{ij} : intersection of row(i) & column(j)

لو عدد الصفوف = عدد الأعمدة

* if $(m = n)$

$\therefore$ the matrix is called "square matrix"

* a_{ii} all i is called "diagonal element"

$tr(A)$ is summation of diagonal element

$$tr(A) = \sum_{i=1}^{n} a_{ii}$$

Ex $A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$ $\therefore Tr(A) = 1 + 2 = \boxed{3}$

note * $Tr(A) = tr(A^t)$

* if $tr(A) =$ Zero
$\therefore$ traceless matrix

from previous example

$A^T = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ $\therefore Tr(A^t) = 1 + 2 = \boxed{3}$

Sum & sub of matrix

شرط الجمع أو الطرح ان المصفوفتين يكونوا ليهم نفس عدد الصفوف والأعمدة

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 7 \\ -1 & 9 \end{pmatrix}$$

$$\therefore A + B = \begin{pmatrix} 3 & 9 \\ 2 & 13 \end{pmatrix}$$

$$\therefore A - B = \begin{pmatrix} -1 & -5 \\ 4 & -5 \end{pmatrix}$$

$$A + B = B + A$$

$$A - B \neq B - A$$

multiplication of matrix

* لو عاوز تضرب المصفوفة × ثابت لازم يتضرب في كل عناصر المصفوفة

$$* \; A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$4A = \begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix}$$

$$AB \neq BA$$

* المصفوفتين

$$A_{m \times n} * B_{n \times k} = C_{m \times k}$$

$$n = n$$

الشرط: صفوف الثاني = أعمدة الأول

وناتج المصفوفة يكون على size $\boxed{m \times k}$

Ex $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}_{2 \times 2}, \quad B = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}_{2 \times 3}$

$$AB = \begin{pmatrix} 5 & 11 & 17 \\ 11 & 25 & 39 \end{pmatrix}$$

transpose of matrix

بدل الأعمدة صفوف والصفوف أعمدة

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \therefore A^t = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

~~A~~

Identity matrix

القطر الرئيسي ب $\boxed{1}$

diagonal

$$\boxed{\begin{aligned} a_{ij} &= 0 \quad \text{if } i \neq j \\ a_{ij} &= 1 \quad \text{if } i = j \end{aligned}}$$

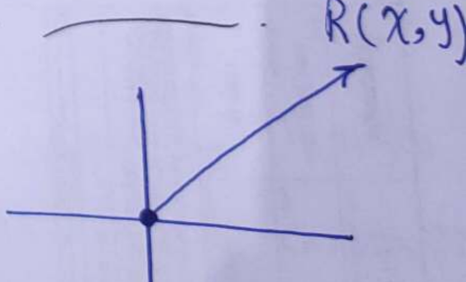
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\boxed{\begin{aligned} AB &= C \\ (AB)_{ij} &= c_{ij} \\ &\sum_{m=1}^{n} A_{im} B_{mj} \end{aligned}} \longrightarrow$$ Code for multiplication of matrix

هام

$$\boxed{\begin{aligned} (AB)^T &= B^T A^T \\ AI &= A\underset{\text{Identity}}{I} = A \\ AA^{-1} &= A^{-1}A = I \end{aligned}}$$

vectors



$A = [a \ b \ c \ d \ e]$ row vector

$B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ Column vector

$R = \begin{bmatrix} x \\ y \end{bmatrix}$

$R = \sqrt{x^2 + y^2}$

$R^T R = \|R\| = |R| = \text{Norm}$

Ex

$$R = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$R^t = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\|R\| = |R| = \sqrt{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \end{bmatrix}} = \sqrt{1+4=5}$$

متجهات ← جبري ←

$$\text{Norm} = \sqrt{5}$$

orthognal matrix A & B row vector

$$AB = BA^t = AB^t = 0$$

Ex

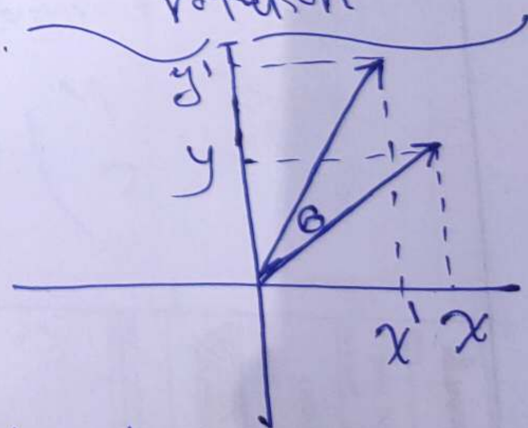
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$AB^t = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

∴ orthognal Vector

orthognal matrix for rotation

$$\begin{pmatrix} \boxed{\cos\theta \quad -\sin\theta} \\ \boxed{\sin\theta \quad \cos\theta} \end{pmatrix} \quad \begin{matrix} \to V_1 \\ \to V_2 \end{matrix}$$



$$V_1 = [\cos\theta \quad -\sin\theta]$$

$$V_2 = [\sin\theta \quad \cos\theta]$$

direction ⇒ Change

magnitude ⇒ Constant

$$V_1 V_2^t = 0 = \sin\theta\cos\theta - \sin\theta\cos\theta$$

Complex

$Z = a + i b$, $(a \,\&\, b) \rightarrow R$

$(i) \rightarrow$ imagine number

$Z^* = a - i b$

conj

$Z_1 \pm Z_2 = (a_1 \pm a_2) \pm i (b_1 \pm b_2)$

موجبة دائماً

$\| Z \| = \sqrt{a^2 + b^2} = Z Z^*$

normalization

$|A| \neq 1 \longrightarrow \| \hat{A} \| = 1$

$A = [1 \quad 2]$

$A A^T = [1 \quad 2] \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5$

$\hat{A} = \frac{1}{\sqrt{\|A\|}} A$

$\hat{A} = \frac{1}{\sqrt{5}} [1 \quad 2] = \left[\frac{1}{\sqrt{5}} \quad \frac{2}{\sqrt{5}}\right]$

$\| \hat{A} \| = \sqrt{\frac{1}{5} + \frac{4}{5}} = \sqrt{\frac{5}{5}} = 1$

$A = \begin{pmatrix} 3+5j & 6 \\ 8j & 7-2j \end{pmatrix}$

$A^* = \begin{pmatrix} 3-5j & 6 \\ -8j & 7+2j \end{pmatrix}$

ضرب المصفوفات لا يجوز في الأعداد المركبة

$A = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

$A^t A = (z_1, z_2) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1^2 + z_2^2$

not real

$A = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$

$A^{*t} A = (Z_1^* \quad Z_2^*) \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} =$

$Z_1 Z_1^* + Z_2 Z_2^* = a_1^2 + b_1^2 + a_2^2 + b$

common factor

$A = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$

$A = 3 \begin{pmatrix} 4/3 & 1 \\ 2/3 & 1/3 \end{pmatrix}$

Special vector

$u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $u_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$u_1^T u_2 = 0$

$\therefore u_1 \perp u_2$

$\|u_1\| = 1$

$\|u_2\| = 1$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$X u_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$X u_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\boxed{X u_1 = u_2}$, $\boxed{X u_2 = u_1}$

$A = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ عبر عن المصفوفة دي ب u_1 , u_2

$A = 3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 5 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$H u_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\boxed{H u_1 = u_1 + u_2}$

$H u_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\boxed{H u_2 = u_1 - u_2}$

finish lecture