Sheet 2

Pb1

In an LTL (less-than-truckload) trucking company, terminal docks include *casual* workers who are hired temporarily to account for peak loads. At the Omaha, Nebraska, dock, the minimum demand for casual workers during the seven days of the week (starting on Monday) is 20, 14, 10, 15, 18, 10, 12 workers. Each worker is contracted to work five consecutive days. Determine an optimal weekly hiring practice of casual workers for the company.

Pb2

On most university campuses students are contracted by academic departments to do errands, such as answering the phone and typing. The need for such service fluctuates during work hours (8:00 A.M. to 5:00 P.M.). In the IE department, the minimum number of students needed is 2 between 8:00 A.M. and 10:00 A.M., 3 between 10:01 A.M. and 11:00 A.M., 4 between 11:01 A.M. and 1:00 P.M., and 3 between 1:01 P.M. and 5:00 P.M. Each student is allotted 3 consecutive hours (except for those starting at 3:01, who work for 2 hours and those who start at 4:01, who work for one hour). Because of their flexible schedule, students can usually report to work at any hour during the work day, except that no student wants to start working at lunch time (12:00 noon). Determine the minimum number of students the IE department should employ and specify the time of the day at which they should report to work.

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Pb3

Universal Mines Inc. operates three mines in West Virginia. The ore from each mine is separated into two grades before it is shipped; the daily production capacities of the mines, as well as their daily operating costs, are as follows:

	High-Grade Ore, tons/day	Low-Grade Ore, tons/day	Operating Cost, $1000/day
Mine I	4	4	20
Mine II	6	4	22
Mine III	1	6	18

Universal has committed itself to deliver 54 tons of high-grade ore and 65 tons of low-grade ore by the end of the week. It also has labor contracts that guarantee employees in each mine a full day's pay for each day or fraction of a day the mine is open. Determine the number of days each mine should be operated during the upcoming week if Universal Mines is to fulfill its commitment at minimum total cost.

Pb4

A town has budgeted $250 000 for the development of new rubbish disposal areas. Seven sites are available, whose projected capacities and development costs are given below. Which sites should the town develop?

Site	A	B	C	D	E	F	G
Capacity, tons/wk	20	17	15	15	10	8	5
Cost, $1000	145	92	70	70	84	14	47

Pb5

A factory manufactures a product each unit of which consists of 5 units of part A and 4 units of part B. The two parts A and B require different raw materials of which 120 units and 240 units respectively are available. These parts can be manufactured by three different methods. Raw material requirements per production run and the number of units for each part produced are given below.

Table 2.2

	Input per run (units)		Output per run (units)	
Method	Raw material 1	Raw material 2	Part A	Part B
1	7	5	6	4
2	4	7	5	8
3	2	9	7	3

Determine the number of production runs for each method so as to maximize the total number of complete units of the final product.

Pb6

A certain farming organization operates three farms of comparable productivity. The output of each farm is limited both by the usable acreage and by the amount of water available for irrigation. Following are the data for the upcoming season:

Farm	Usable acreage	Water available in acre feet
1	400	1,500
2	600	2,000
3	300	900

The organization is considering three crops for planting which differ primarily in their expected profit per acre and in their consumption of water. Furthermore, the total acreage that can be devoted to each of the crops is limited by the amount of appropriate harvesting equipment available.

Crop	Minimum acreage	Water consumption in acre feet per acre	Expected profit per acre
A	700	5	Rs. 400
B	800	4	Rs. 300
C	300	3	Rs. 100

In order to maintain a uniform work load among the farms, it is the policy of the organization that the percentage of the usable acreage planted must be the same at each farm. However, any combination of the crops may be grown at any of the farms. The organization wishes to know how much of each crop should be planted at the respective farms in order to maximize expected profit Formulate this as a linear programming problem.

Pb7

A manufacturer has five lathes and three milling machines in his workshop and produces an assembly that consists of 2 units of part A and 3 units of part B. The processing time for each part on the two types of machines is given below.

Table 2.3

Part	Processing time in minutes on a	
	Lathe	Milling machine
A	10	18
B	25	12

In order to maintain a uniform work-load on the two types of machines, the manufacturer has framed a policy that no type of machine should run more than 40 minutes per day longer than the other machine. Formulate the problem as L.P. problem if the objective is to produce a maximum number of assemblies in any 8-hour working day.

Pb8

A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given

Food type	Yield per unit			cost/unit
	Proteins	Fats	Carbohydrates	(Rs.)
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate the L.P. model for the problem.

Pb9

A paper mill produces rolls of paper used in cash registers. Each roll of paper is 100m in length and can be used in widths of 2, 4, 6 and 10 cm. The company's production process results in rolls that are 24 cm in width. Thus the company must cut its 24 cm wide roll to the desired widths. It has six cutting alternatives as follows :

Cutting alternative	Width of rolls (cm)				Waste (cm)
	2	4	6	10	
1	6	3	—	—	—
2	—	3	2	—	—
3	1	1	1	1	2
4	—	—	2	1	2
5	—	4	1	—	2
6	4	2	1	—	2

The minimum demand for the four rolls is as follows :

Roll width (cm)	Demand
2	2,000
4	3,600
6	1,600
10	500

The paper mill wishes to minimize the waste resulting from trimming to size. Formulate the L.P. model.

Pb10

A firm manufactures three products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 respectively. The firm has two machines and the required processing time in minutes for each machine on each product is given below.

Table 2.15

Machine	Product A	Product B	Product C
C_1	4	3	5
D_1	2	2	4

Machines C_1 and D_1 have 2,000 and 2,500 machine-minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but no more than 150 A's. Set up an L.P. model to maximize the profit.

Pb11

A firm can produce three types of cloth, say, A, B and C. Three kinds of wool are required for it, say, red wool, green wool and blue wool. One unit length of type A cloth needs 2 yards of red wool and 3 yards of blue wool; one unit length of type B cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool; and one unit length of type C cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has a stock of only 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3, of type B cloth is Rs. 5 and that of type C cloth is Rs. 4. Formulate the problem as linear programming problem.

Pb12

A dairy feed company may purchase and mix one or more of the three types of grains containing different amounts of nutritional elements. The data are given in the table below.

The production manager specifies that any feed mix for his livestock must meet at least minimal nutritional requirements and seeks the least costly among all such mixes.

Table 2.17

	Item	*One unit weight of*			*Minimal requirement*
		Grain 1	*Grain 2*	*Grain 3*	
Nutritional ingredients	A	2	3	7	1,250
	B	1	1	0	250
	C	5	3	0	900
	D	6	25	1	1232.5
Cost/unit weight (Rs.)		41	35	96	

Analyse the situation to recognize the key decision, objective, alternatives and restrictions. Formulate linear programming model for the problem.

Pb13

A farmer has a 100-acre farm. He can sell all the tomatoes, lettuce or radishes he can raise. The price he can obtain Rs. 1 per kg for tomatoes, Rs. 0.75 a head for lettuce and Rs. 2 per kg for radishes. The average yield per acre is 2,000 kg of tomatoes, 3,000 heads of lettuce and 1,000 kg of radishes. Fertilizer is available at Re. 0.50 per kg and the amount required per acre is 100 kg each for tomatoes and lettuce and 50 kg for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at Rs. 20 per man-day.

Formulate an L.P. model for this problem in order to maximize the farmer's total profit.

Pb14

A truck company requires the following number of drivers for its trucks during 24 hours :

Time	*No. required*
00—04 hr.	5
04—08 hr.	10
08—12 hr.	20
12—16 hr.	12
16—20 hr.	22
20—24 hr.	8

According to the shift schedule a driver may join for duty at midnight, 04, 08, 12, 16, 20 hours and work continuously for 8 hours.

Pb15

A fruitseller mixes pears, apples and grapes to make three different types of baskets. Pears are purchased at a cost of Rs. 4 a kg, apples at Rs. 6 a kg and grapes at Rs. 10 a kg. The specifications for the three types of baskets are as follows :

Type 1 must have at least 40% pears and at the most 25% grapes,
type 2 must have at least 30% pears and at the most 40% grapes, and
type 3 offers no restrictions on the constituents.

The sale price of type 1 is Rs. 7 per kg, of type 2 is Rs. 10 per kg and of type 3 is Rs. 6 per kg. The daily supply of fruit is limited to 80 kg of pears, 120 kg of apples and 40 kg of grapes. Formulate the L.P. model to represent how the fruit be mixed to maximize profit.

Pb16

The Transportation Problem: A manufacturer of sheet polyethylene has two plants, one located in Salt Lake City and the other located in Denver. There are three distributing warehouses, one in Los Angeles, another in Chicago, and the third in New York City. The Salt Lake City plant can supply 120 tons of the product per week, whereas the Denver plant can supply 140 tons of the product per week. The Los Angeles warehouse needs 100 tons weekly to meets its demand, the Chicago warehouse needs 60 tons weekly, and the New York City warehouse needs 80 tons weekly. The following table gives the shipping cost (in dollars) per ton of the product:

		To		
		Los Angeles	Chicago	New York City
From	Salt Lake City	5	7	9
	Denver	6	7	10

How many tons of polyethylene should be shipped from each plant to each warehouse to minimize the total shipping cost while meeting the demand?

Pb17

A local manufacturing firm produces four different metal products, each of which must be machined, polished, and assembled. The specific time requirements (in hours) for each product are as follows.

	Machining, h	Polishing, h	Assembling, h
Product I	3	1	2
Product II	2	1	1
Product III	2	2	2
Product IV	4	3	1

The firm has available to it on a weekly basis 480 h of machine time, 400 h of polishing time, and 400 h of assembly time. The unit profits on the products are \$6, \$4, \$6, and \$8, respectively. The firm has a contract with a distributor to provide 50 units of product I and 100 units of any combination of products II and III each week. Through other customers, the firm can sell each week as many units of products I, II, and III as it can produce, but only a maximum of 25 units of product IV. How many units of each product should the firm manufacture each week to meet all contractual obligations and maximize its total profit? Assume that any unfinished pieces can be completed the following week.

Pb18

A caterer must prepare from five fruit drinks in stock 500 gal of a punch containing at least 20 percent orange juice, 10 percent grapefruit juice, and 5 percent cranberry juice. If inventory data are as shown below, how much of each fruit drink should the caterer use to obtain the required composition at minimum total cost?

	Orange Juice, %	Grapefruit Juice, %	Cranberry Juice, %	Supply, gal	Cost, \$/gal
Drink A	40	40	0	200	1.50
Drink B	5	10	20	400	0.75
Drink C	100	0	0	100	2.00
Drink D	0	100	0	50	1.75
Drink E	0	0	0	800	0.25

Pb19

A semiconductor corporation produces a particular solid-state module that it supplies to four different television manufacturers. The module can be produced at each of the corporation's three plants, although the costs vary because of differing production efficiencies at the plants. Specifically, it costs \$1.10 to produce a module at plant A, \$0.95 at plant B, and \$1.03 at plant C. Monthly production capacities of the plants are 7500, 10 000, and 8100 modules, respectively. Sales forecasts project monthly demand at 4200, 8300, 6300, and 2700 modules for television manufacturers I, II, III, and IV, respectively. If the cost (in dollars) for shipping a module from a factory to a manufacturer is as shown below, find a production schedule that will meet all needs at minimum total cost.

	I	II	III	IV
A	0.11	0.13	0.09	0.19
B	0.12	0.16	0.10	0.14
C	0.14	0.13	0.12	0.15

Pb 20

A legal firm has accepted five new cases, each of which can be handled adequately by any one of its five junior partners. Due to differences in experience and expertise, however, the junior partners would spend varying amounts of time on the cases. A senior partner has estimated the time requirements (in hours) as shown below:

	Case 1	Case 2	Case 3	Case 4	Case 5
Lawyer 1	145	122	130	95	115
Lawyer 2	80	63	85	48	78
Lawyer 3	121	107	93	69	95
Lawyer 4	118	83	116	80	105
Lawyer 5	97	75	120	80	111

Determine an optimal assignment of cases to lawyers such that each junior partner receives a different case and the total hours expended by the firm is minimized.

Pb21

Recreational Motors manufactures golf carts and snowmobiles at its three plants. Plant A produces 40 golf carts and 35 snowmobiles daily; plant B produces 65 golf carts daily, but no snowmobiles; plant C produces 53 snowmobiles daily, but no golf carts. The costs of operating plants A, B, and C are respectively $210 000, $190 000, and $182 000 per day. How many days (including Sundays and holidays) should each plant operate during September to fulfill a production schedule of 1500 golf carts and 1100 snowmobiles at minimum cost? Assume that labor contracts require that once a plant is opened, workers must be paid for the entire day.

Pb22

The manager of a supermarket meat department finds she has 200 lb of round steak, 800 lb of chuck steak, and 150 lb of pork in stock on Saturday morning, which she will use to make hamburger meat, picnic patties, and meat loaf. The demand for each of these items always exceeds the supermarket's supply. Hamburger meat must be at least 20 percent ground round and 50 percent ground chuck (by weight); picnic patties must be at least 20 percent ground pork and 50 percent ground chuck; and meat loaf must be at least 10 percent ground round, 30 percent ground pork, and 40 percent ground chuck. The remainder of each product is an inexpensive nonmeat filler which the store has in unlimited supply. How many pounds of each product should be made if the manager desires to minimize the amount of meat that must be stored in the supermarket over Sunday?

Pb23

A pet store has determined that each hamster should receive at least 70 units of protein, 100 units of carbohydrates, and 20 units of fat daily. If the store carries the six types of feed shown in Table 1-3, what blend of feeds satisfies the requirements at minimum cost to the store?

Table 1-3

Feed	Protein, units/oz	Carbohydrates, units/oz	Fat, units/oz	Cost, ¢/oz
A	20	50	4	2
B	30	30	9	3
C	40	20	11	5
D	40	25	10	6
E	45	50	9	8
F	30	20	10	8

Pb24

A 400-meter medley relay involves four different swimmers, who successively swim 100 meters of the backstroke, breaststroke, butterfly, and freestyle. A coach has six very fast swimmers whose expected times (in seconds) in the individual events are given in Table 1-1.

Table 1-1

	Event 1 (backstroke)	Event 2 (breaststroke)	Event 3 (butterfly)	Event 4 (freestyle)
Swimmer 1	65	73	63	57
Swimmer 2	67	70	65	58
Swimmer 3	68	72	69	55
Swimmer 4	67	75	70	59
Swimmer 5	71	69	75	57
Swimmer 6	69	71	66	59

How should the coach assign swimmers to the relay so as to minimize the sum of their times?