OPERATIONAL RESEARCH

Section 7

Section 7: Transportation Model

- Objective: To minimize the transportation cost.
- The values in the table cells represents the cost of transportation between different sources and different destinations.

- Decision Variables: X_{ij}
  ↳ No. of items transferred from source "I" to destination "J"

- Objective Function: $Z = \sum C_{ij} \cdot X_{ij}$

- The problem should be Balanced to start solving the problem.

To make it Balanced

→ Add Dummy Destination "If Demand < Supply"
→ Add Dummy Source "If Supply < Demand"

To Get IBFS, there is 3 ways to solve:

1. North west Corner method
2. Least Cost method
3. Vogel method.

Problem 1

	D1	D2	D3	
S1	5	1	0	20
S2	3	2	4	10
S3	7	5	2	15
S4	9	6	0	15
Demand	5	10	15	

1) Balanced Condition

→ Total Supply = 60

→ Total Demand = 30

Supply > Demand

Add "Dummy Destination" with demand value = 60 - 30 = 30

→ Dummy Destination

	D1	D2	D3	D4	
S1	5	1	0	0	20
S2	3	2	4	0	10
S3	7	5	2	0	15
S4	9	6	0	0	15
Demand	5	10	15	30	

Using North west method:

Start

	D1	D2	D3	D4		
S1	5				$\boxed{20}$	15
S2					10	
S3					15	
S4					15	
	$\boxed{5}$	10	15	30		
	0					

	D1	D2	D3	D4	
S1	5	10			~~15~~ $\boxed{5}$
S2					10
S3					15
S4					15
	0	~~10~~ $\boxed{0}$	15	30	

	D1	D2	D3	D4	
S1	5	10	5		~~5~~ $\boxed{0}$
S2					10
S3					15
S4					15
	0	0	~~15~~ $\boxed{10}$	30	

	D1	D2	D3	D4	
S1	5	10	5		0
S2			10		~~10~~ $\boxed{0}$
S3					15
S4					15
	0	0	~~10~~ $\boxed{0}$	30	

هنا هشطب يا Col
يا Row ← مش فارقة

	D1	D2	D3	D4	
S1	5	10	5		0
S2			10		0
S3			0		15
S4					15
	0	0	0	30	

	D1	D2	D3	D4	
S1	5	10	5		0
S2			10		0
S3			0	15	~~15~~ $\boxed{0}$
S4					15
	0	0	0	~~30~~ $\boxed{15}$	

	D1	D2	D3	D4	
S1	5	10	5		0
S2			10		0
S3			0	15	0
S4				15	~~15~~ $\boxed{0}$
	0	0	0	~~15~~ $\boxed{0}$	

لازم اوصل ان ال
(Demand = Supply = 0)

$$Z = (5 \times 5) + (10 \times 1) + (5 \times 0) + (10 \times 4) + (0 \times 2) + (15 \times 0) + (15 \times 0)$$
$$= \underline{\underline{75}}$$

Note → North west method is the worst as we choose the cell whatever its cost which is wrong

- Using "Vogel" Method:

- Steps of solution:
1. Diff. between 2 least Costs in Row/Col
2. Slect max. diff
3. Allocate in least Cost cell.
4. if ∃ only one Source or only one destination allocate in least Cost cell.

	D1	D2	D3	D4		
S1	[5]	[1]	[0]	[0]	20	0
S2	[3]	[2]	[4]	[0] 10	~~10~~ $\boxed{0}$	$\boxed{2}$ → Max
S3	[7]	[5]	[2]	[0]	15	2
S4	[9]	[6]	[0]	[0]	15	0
	5	10	15	~~30~~ $\boxed{20}$		
	2	1	0	0		

	D1	D2	D3	D4		
S1	[5]	[1] 10	[0]	[0]	~~20~~ $\boxed{10}$	0
S2	[3]	[2]	[4]	[0] 10	0	X
S3	[7]	[5]	[2]	[0]	15	2
S4	[9]	[6]	[0]	[0]	15	0
	5	~~10~~ $\boxed{0}$	15	20		
	2	$\boxed{4}$	0	0		

	D1	D2	D3	D4		
S1	$\boxed{5}$ 5	$\boxed{1}$ 10	$\boxed{0}$	$\boxed{0}$	~~15~~ $\boxed{5}$	0
S2	$\boxed{3}$	$\boxed{2}$	$\boxed{4}$	$\boxed{0}$ 10	0	X
S3	$\boxed{7}$	$\boxed{5}$	$\boxed{2}$	$\boxed{0}$	15	2
S4	$\boxed{9}$	$\boxed{6}$	$\boxed{0}$	$\boxed{0}$	15	0
	~~5~~ $\boxed{0}$	0	15	20		
	$\boxed{2}$	X	0	0		

	D1	D2	D3	D4		
S1	$\boxed{5}$ 5	$\boxed{1}$ 10	$\boxed{0}$	$\boxed{0}$	5	0
S2	$\boxed{3}$	$\boxed{2}$	$\boxed{4}$	$\boxed{0}$ 10	0	X
S3	$\boxed{7}$	$\boxed{5}$	$\boxed{2}$	$\boxed{0}$ 15	~~15~~ $\boxed{0}$	$\boxed{2}$
S4	$\boxed{9}$	$\boxed{6}$	$\boxed{0}$	$\boxed{0}$	15	0
	0	0	15	~~20~~ $\boxed{5}$		
	X	X	0	0		

	D1	D2	D3	D4		
S1	$\boxed{5}$ 5	$\boxed{1}$ 10	$\boxed{0}$ 5	$\boxed{0}$	~~5~~ $\boxed{0}$	$\boxed{0}$
S2	$\boxed{3}$	$\boxed{2}$	$\boxed{4}$	$\boxed{0}$ 10	0	X
S3	$\boxed{7}$	$\boxed{5}$	$\boxed{2}$	$\boxed{0}$ 15	0	X
S4	$\boxed{9}$	$\boxed{6}$	$\boxed{0}$	$\boxed{0}$	15	0
	0	0	~~15~~ $\boxed{10}$	5		
	X	X	0	0		

	D1	D2	D3	D4		
S1	[5] 5	[1] 10	[0] 5	[0]	0	X
S2	[3]	[2]	[4]	[0] 10	0	X
S3	[7]	[5]	[2]	[0] 15	0	X
S4	[9]	[6]	[0] 10	[0]	~~15~~ 5	$\boxed{0}$
	0	0	$\boxed{0}$ 10	5		
	X	X	0	0		

	D1	D2	D3	D4		
S1	[5] 5	[1] 10	[0] 5	[0]	0	X
S2	[3]	[2]	[4]	[0] 10	0	X
S3	[7]	[5]	[2]	[0] 15	0	X
S4	[9]	[6]	[0] 10	[0] 5	~~5~~ $\boxed{0}$	0
	0	0	0	~~5~~ $\boxed{0}$		
	X	X	0	$\boxed{0}$		

$$Z = (5 * 5) + (1 * 10) + (0 * 5) + (0 * 10) + (0 * 15) + (0 * 10) + (0 * 5)$$
$$= 25 + 10 = 35$$

Basic variables = # Sources + # Destinations - 1

- Steps of Solutions: "Optimal Solution"

1. For B.V → $C_{ij} = U_i + V_j$
2. For Non-B.V → $C_{ij} - U_i - V_j$
3. If all "$C_{ij} - U_i - V_j$" ≥ 0 → Optimal
   else, Select most Negative value "EV" → بيكون هو بداية الloop ونهايتها
4. Loop.

- Conditions of loop:
→ Horizontal and vertical lines.
→ Start and end EV
→ least no. of cells = 4
→ No 3 cells Consecutive in Row/Col.
→ # cells should be even.

EV: Non-Basic to Basic
↳ So, we add +ve sign.

Example 1

	D1	D2	D3	D4	Supply
S1	[0]	[2] 15	[20]	[11] 0	15
S2	[12]	[7]	[9] 15	[20] 10	25
S3	[4] 5	[14]	[16]	[18] 5	10
	5	15	15	15	

	D1	D2	D3	D4	Ui
S1	[0] (0-11+14) = (3)	[2] 15	[20] (20-11+11) = (20)	[11] 0	11-0=11
S2	[12] (12-20+14) = (6)	[7] (7-20+9) = (-4)	[9] 15	[20] 10	→ 20-0=20 "Cij=Ui+Vj"
S3	[4] 5	[14] (14-18+9) = (5)	[16] (16-18+11) = (9)	[18] $\boxed{5}$	→ 18-0=18
Vj	-14	2-11=-9	-11	0	

	D1	D2	D3	D4	Ui
S1	[0] (3)	[2] 15 ⊖	[20] (20)	[11] ⊕ 0	11
S2	[12] (6)	[7] ⊕ (-4) E.V	[9] 15	[20] ⊖ 10	20
S3	[4] 5	[14] (5)	[16] (9)	[18] 5	18
Vj	-14	-9	-11	0	

- Loop = 4 cells
- EV. = -4

Donors = 15, 10 → Min(15, 10) = 10

So, 10 is DV.

	D1	D2	D3	D4	u_i
S1	10	2 15-10 = 5	20	11 0+10 = 10	11
S2	12	7 10	9 15	20	20
S3	4 5	14	16	18 5	18
v_j	-14	-9	-11	0	

- Another example: "Get Optimal Solution"

	D1	D2	D3	D4	u_i
S1	[5] (−) 5	[11] 10	[8] 5 (+)	[6] (0)	0-0=0
S2	[3] (+) (-2)	[9] (1)	[4] (4)	[8] (−) 10	0-0=0
S3	[7] (2)	[8] (4)	[2] (2)	[6] 15	0-0=0
S4	[9] (4)	[6] (5)	[8] (−) 10	[8] 5 (+)	0-0=0
v_j	5-0=5	1-0=1	0-0=0	0	

Entry Variable = -2

Donors "-ve" = (5, 10, 10)
↳ Minimum = 5 → Depart

depart

	D1	D2	D3	D4
S1	[5] 0	[11] 10	[8] 10	[6] (0)
S2	[3] 5	[9] (1)	[4] (4)	[8] 5
S3	[7] (2)	[8] (4)	[2] (2)	[6] 15
S4	[9] (4)	[6] (5)	[8] 5	[8] 10

entry