# OPERATIONAL RESEARCH

Section 6

# Dual Simplex Method.

_ Dual Simplex Method → بيحافظ على الـ Optimality
↳ Efficient but not powerful
بيحل بسرعة بس مش أي مسألة

_ Regular Simple Method → بيحافظ على الـ Feasibility
↳ Powerful but not efficient
بيحل أي مسألة بس مش بسرعة

_ Dual Simplex Method: → بنجيب الأول الـ DV وبعدين الـ EV

→ Conditions to use:
if max: Z = — — — —
if min: Z = + + + +

→ To Solve:
1. all vars $\geq 0$
2. all Const. $\leq 0$ → Then "=" by adding Slack
3. Some R.H.S $\leq 0$

→ Algorithms:
↳ we will start with "Optimal Sol." but not Feasible "All slack var = R.H.S"

1. Feasibility Test: Select from R.H.S "most -ve" to be "D.V"
لو كله +ve يبقى خلاص وصلنا للـ Feasible Sol.

2. Optimality Test: For each -ve value in the D.V. calculate the abs. ratio → E.V "Min Ratio"
if none → NFS: stop

3. Make row operations to make Col. of E.V as identity.

**Example 1:**

Max: $Z = -4X_1 - 3X_2$

Sub. to:

$X_1 + X_2 \geq 2$

$X_1 + 2X_2 \geq 8$

all vars $\geq 0$

**Solution:**

1- Condition of DSM → Achieved.

2- To Solve:

→ All vars $\geq 0$

→ All Cons. $\leq$

$X_1 + X_2 \geq 2 \rightarrow -X_1 - X_2 \leq -2$

$X_1 + 2X_2 \geq 8 \rightarrow -X_1 - 2X_2 \leq -8$

all vars $\geq 0$

So,

max: $Z = -4X_1 - 3X_2 + 0S_1 + 0S_2$

Sub. to:

$-X_1 - X_2 + S_1 = -2$

$-X_1 - 2X_2 + S_2 = -8$

all vars $\geq 0$

Initial Sol: $S_1 = -2$, $S_2 = -8$, $X_1 = X_2 = 0$

| | | $X_1$ | $X_2$ | $S_1$ | $S_2$ | |
|---|---|---|---|---|---|---|
| | | -4 | -3 | 0 | 0 | |
| $S_1$ | 0 | -1 | -1 | 1 | 0 | -2 |
| $S_2$ | 0 | -1 | -2 | 0 | 1 | -8 → Most -ve |
| $E_j$ | | 0 | 0 | 0 | 0 | |
| $E_j - C_j$ | | 4 | 3 | 0 | 0 | |

- Absolute Ratio: →

$\left|\frac{4}{-1}\right| = 4$ and $\left|\frac{3}{-2}\right| = 1.5$

So,

D.V : Most -ve "S2"
E.V : Min Ratio "x2"

- Row Operations:

→ $R_2 * (-1/2)$
$R_1 + R_2$

| | | X1 | X2 | S1 | S2 | |
|---|---|---|---|---|---|---|
| | | -4 | -3 | 0 | 0 | |
| S1 | 0 | -1/2 | 0 | 1 | -1/2 | 2 |
| X2 | -3 | 1/2 | 1 | 0 | -1/2 | 4 |
| Ej | | 3/2 | 3 | 0 | -3/2 | |
| Ej-Cj | | 5/2 | 0 | 0 | 3/2 | |

Feasible Sol.

Optimal Feasible Sol. :

→ $X_1 = 0$
$X_2 = 4$
$S_1 = 2$
$S_2 = 0$

↳ $Z = -4(0) - 3(4) = -12$

# Section 6: Integer Programing.

* For example: $X_1, X_2 \geq 0$ and integer

الأول هنحل عادي على شرط الـ Non_Neg. و هنتجاهل شرط الـ integer.

هنحل بالـ RSM أو الـ SM أو الـ DSM وبعد ما نجيب optimal feasible Sol. نبدأ نحل مشكلة الـ integer

"Final Tableau"

| | | |
|---|---|---|
| $X_1$ | | 2 |
| $X_2$ | | 4 |
| $S_1$ | | 5 |
| | optimal | |

Feasible $\geq 0$ and Integer

"Final Tableau"

| | | |
|---|---|---|
| $X_1$ | | 2 1/2 |
| $X_2$ | | 4 1/3 |
| $S_1$ | | 5 |

Feasible $\geq 0$ and Not Integer

Our work will be on "Final Tableau"

* There are 2 methods to solve the integer problems

Rounding Method

$\epsilon$-close Method.

Example: Max: $Z = 3x_1 + 4x_2$

s.t:

$2x_1 + x_2 \le 6$

$2x_1 + 3x_2 \le 9$

all variables $\geq 0$ and integers.

Final Tableaus

| | | $x_1$ | $x_2$ | $S_1$ | $S_2$ | |
|---|---|---|---|---|---|---|
| | | 3 | 4 | 0 | 0 | |
| $x_1$ | 3 | 1 | 0 | 3/4 | -1/4 | 9/4 |
| $x_2$ | 4 | 0 | 1 | -1/2 | 1/2 | 3/2 |
| | | 0 | 0 | 1/4 | 5/4 | $\frac{51}{4} = 12.75$ |

$\geq 0$ but not integer.

Solution:

1- Rounding Method:

$x_1 = \frac{9}{4} = 2.25$ → Round up $x_1 = 3$ / → Round down $x_1 = 2$

$x_2 = \frac{3}{2} = 1.5$ → Round up $x_1 = 2$ / → Round down $x_2 = 1$

Then, we have 4 possibilities:

(3,2) (3,1)

(2,2) (2,1)

1- $(3,2) \rightarrow 2(3) + 2 = 8 \le 6$ X

Not Solution as it didn't achieve 1st Const.

2. $(3,1) \rightarrow 2(3) + 1 = 7 \le 6$ X

Not Solution as it didn't achieve 1st Const.

3. $(2,2) \to 2(2)+2 \leq 6$ ✓

$2(2)+3(2) = 10 \leq 9$ ✗

Not Solution as it didn't achieve the 2nd Const

4. $(2,1) \to 2(2)+1 = 5 \leq 6$ ✓

$2(2)+3(1) = 7 \leq 9$ ✓

So, $Z = 3(2)+4(1) = 10$

"Optimal Feasible Integer Solution"

2. $\epsilon$-close Method:

Let: $Z = f(x)$ and $Z_f$ = Value of $Z$

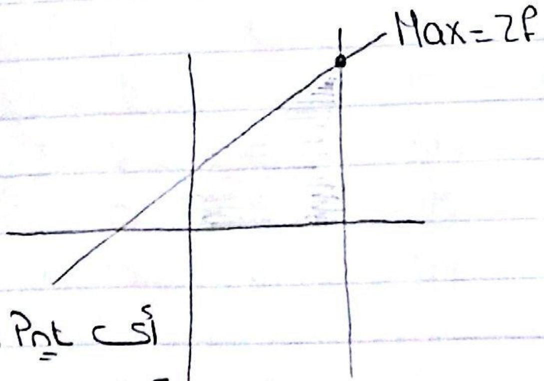
and add new Constraint

$(\alpha . f(x) \leq \alpha Z_f - \epsilon)$

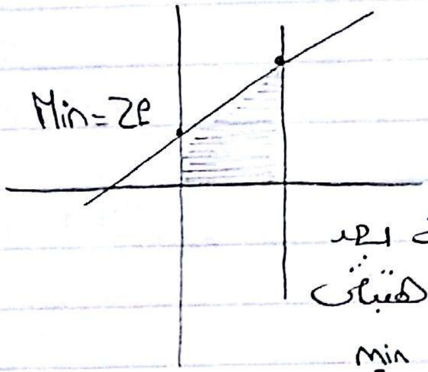
$$\alpha = \begin{cases} 1 \to \text{for max} \to f(x) \leq Z_f - \epsilon \\ -1 \to \text{for min} \to -f(x) \leq -Z_f - \epsilon \end{cases}$$

$\downarrow$ mult. (-1)

$f(x) \geq Z_f + \epsilon$




أي Pnt بعد كده جوه هتبقى أقل من الـ Max




أي نقطة بعد كده جوه هتبقى اكبر من الـ Min

Where, $\epsilon$ is non-negative very small real number.