



OPERATIONAL RESEARCH

Section2



Problem 3 in sheet 2:

Universal Mines Inc. operates three mines in West Virginia. The ore from each mine is separated into two grades before it is shipped; the daily production capacities of the mines, as well as their daily operating costs, are as follows:

	High-Grade Ore, tons/day	Low-Grade Ore, tons/day	Operating Cost, \$1000/day
Mine I	4	4	20
Mine II	6	4	22
Mine III	1	6	18

Universal has committed itself to deliver 54 tons of high-grade ore and 65 tons of low-grade ore by the end of the week. It also has labor contracts that guarantee employees in each mine a full day's pay for each day or fraction of a day the mine is open. Determine the number of days each mine should be operated during the upcoming week if Universal Mines is to fulfill its commitment at minimum total cost.

	High Grade (Ton/day)	Low Grade (ton/day)	Operating Cost (\$1000/day)
Mine 1	4	4	20
Mine 2	6	4	22
Mine 3	1	6	18
Demand	54 ton/week	65 ton/week	

*Has labor contracts that guarantee employees in each mine a full day's pay for each day or fraction of a day the mine is open

Determine # of days should be operated to fulfill its commitment during upcoming week at min cost?

Solution:

Decision Variables:

- x_1 → # of days for mine 1/week
- x_2 → # of days for mine 2/week
- x_3 → # of days for mine 3/week

Objective: Minimize $Z = 20x_1 + 22x_2 + 18x_3$

Subject to:

$$4x_1 + 6x_2 + x_3 \geq 54 \text{ tons}$$

$$4x_1 + 4x_2 + 6x_3 \geq 65 \text{ tons}$$

$$x_1, x_2, x_3 \geq 0 \text{ and } x_1, x_2, x_3 \leq 7$$

x_1, x_2, x_3 are integers

على ان تكون كل من المتغيرات
Full day's pay for each day or
Fraction of day the mine is
open

Problem 4 in Sheet 2.

A town has budgeted \$250,000 for the development of new rubbish disposal areas. Seven sites are available whose projected capacities and development costs are given below. Which sites should the town develop?

Site	A	B	C	D	E	F	G
Capacity, tons/wk	20	17	15	15	10	8	5
Cost, \$1000	145	92	70	70	84	14	47

- Budget : \$ 250,000

Sites	A	B	C	D	E	F	G
Capacity (ton/week)	20	17	15	15	10	8	5
Cost (1000 \$)	145	92	70	70	84	14	47

- Which sites should the town develop?

أنا صحت محتاج أقل ال Cost لأن 5 و 5 و 5 و 5 و 5 و 5 و 5
لأن إزود عن ال tons

- Decision variables:

$$x_i ; 1 \leq i \leq 7$$

→ 1 : Site should be developed

→ 0 : Otherwise.

بلا هات ال Site ، بلا صحت لازم

- Objective:

$$\text{Maximize Capacity } z: 20x_1 + 17x_2 + 15x_3 + 15x_4 + 10x_5 + 8x_6 + 5x_7$$

- Subject to:

$$145x_1 + 92x_2 + 70x_3 + 70x_4 + 84x_5 + 14x_6 + 47x_7 \leq 250$$

where: $x_i \geq 0$

→ Binary, Integers.

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Problem 6 in sheet 2:

A certain farming organization operates three farms of comparable productivity. The output of each farm is limited both by the usable acreage and by the amount of water available for irrigation. Following are the data for the upcoming season:

Farm	Usable acreage	Water available in acre feet
1	400	1,500
2	600	2,000
3	300	900

The organization is considering three crops for planting which differ primarily in their expected profit per acre and in their consumption of water. Furthermore, the total acreage that can be devoted to each of the crops is limited by the amount of appropriate harvesting equipment available.

Crop	Minimum acreage	Water consumption in acre feet per acre	Expected profit per acre
A	700	5	Rs. 400
B	800	4	Rs. 300
C	300	3	Rs. 100

In order to maintain a uniform work load among the farms, it is the policy of the organization that the percentage of the usable acreage planted must be the same at each farm. However, any combination of the crops may be grown at any of the farms. The organization wishes to know how much of each crop should be planted at the respective farms in order to maximize expected profit. Formulate this as a linear programming problem.

Farm	Usable acreage	Water available in acre feet
1	400	1,500
2	600	2,000
3	300	900

Crop	min acreage	Water Cons. per acr.	Profit per acr.
A	700	5	400
B	800	4	300
C	300	3	100

* Percentage of usable acreage planted must be the same at any of farms.

* Any combination of the crops may be grown at any farm.

* How much of each crop should be planted at the respective farms to maximize the profit.

Solution:

Decision Variables:

$x_{1A}, x_{1B}, x_{1C}, x_{2A}, x_{2B}, x_{2C}, x_{3A}, x_{3B}, x_{3C}$

$x_{ij} : 1 \leq i \leq 3 \rightarrow \text{Farm}$

$A \in j \in C \rightarrow \text{Crop}$
 $\rightarrow \text{Area planted from } j \text{ in Farm } i$

objective:

Maximize Profit:

$$Z = 400(x_{1A} + x_{2A} + x_{3A})$$

$$+ 300(x_{1B} + x_{2B} + x_{3B})$$

$$+ 100(x_{1C} + x_{2C} + x_{3C})$$

Subject to:

$$\rightarrow x_{1A} + x_{1B} + x_{1C} \leq 400 \quad * \text{No. of area. in each Farm.}$$

$$x_{2A} + x_{2B} + x_{2C} \leq 600$$

$$x_{3A} + x_{3B} + x_{3C} \leq 300$$

$$\rightarrow 5x_{1A} + 4x_{1B} + 3x_{1C} \leq 1500 \quad * \text{water available in each Farm.}$$

$$5x_{2A} + 4x_{2B} + 3x_{2C} \leq 2000$$

$$5x_{3A} + 4x_{3B} + 3x_{3C} \leq 900$$

$$\rightarrow x_{1A} + x_{2A} + x_{3A} \geq 700 \quad * \text{Min. area. for each crop}$$

$$x_{1B} + x_{2B} + x_{3B} \geq 800$$

$$x_{1C} + x_{2C} + x_{3C} \geq 300$$

Percentage:

$$\frac{x_{1A} + x_{1B} + x_{1C}}{400} = \frac{x_{2A} + x_{2B} + x_{2C}}{600} = \frac{x_{3A} + x_{3B} + x_{3C}}{300}$$

$$x_{ij} \geq 0$$