Operations Research

Lecture 3:

Linear Programming: Mathematical Models (Part II)

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Example 9: Trim Loss Problem

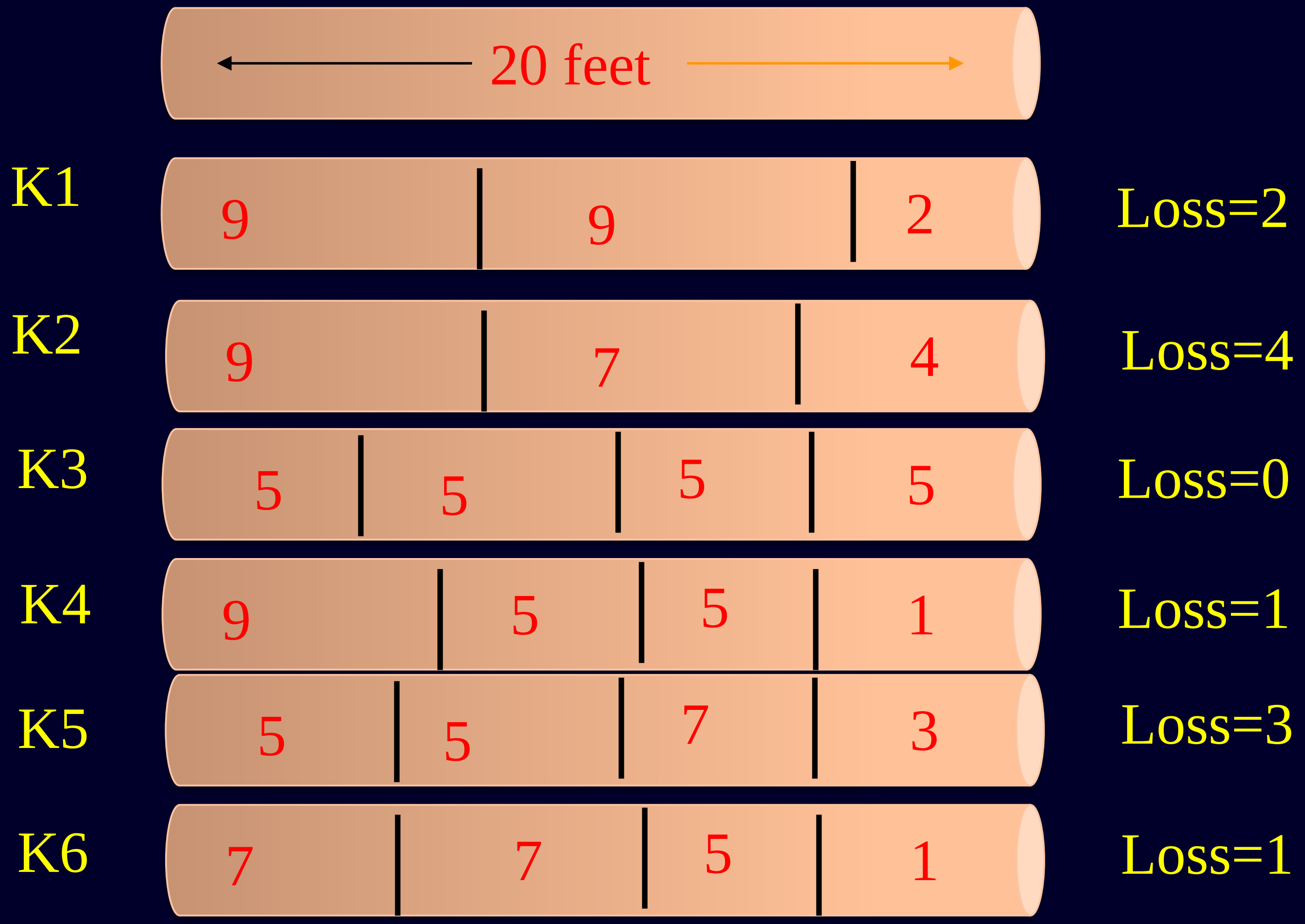
Rolls of paper having a fixed length and width 20 feet are being manufactured by a paper company. These rolls have to be cut with different knife setting to satisfy the following demand:

width	9	7	5
Min Number of rolls:	200	120	450

Obtain the linear programming formulation of the problem to determine the cutting pattern, so that the demand is satisfied and wastage of paper is minimized (Formulate only the linear problem model)

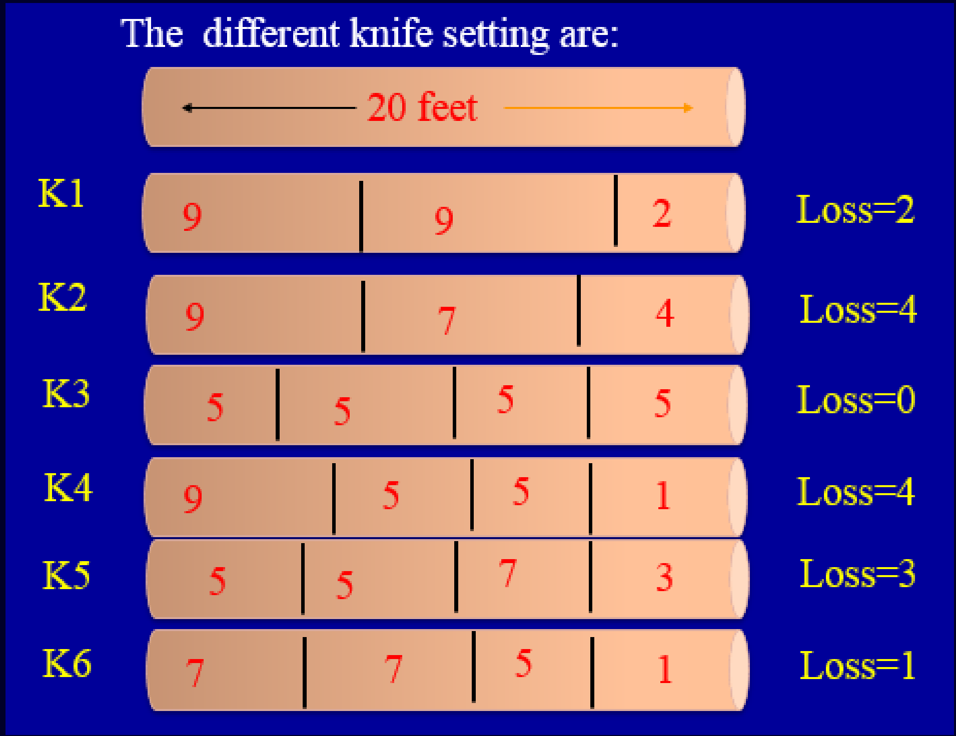
Trim Loss Model

The different knife setting are:



Trim loss Model cont…

	K1	K2	K3	K4	K5	K6
Width 5	0	0	4	2	2	1
Width 7	0	1	0	0	1	2
Width 9	2	1	0	1	0	0
Loss	2	4	0	1	3	1



Let x_1 the number of rolls that will be cut by knife setting K1

Let x_i the number of rolls that will be cut by knife setting Ki, i=1,2…6

Minimize: $Z = 2x_1 + 4x_2 + 0x_3 + x_4 + 3x_5 + x_6$

Subject to: $4x_3 + 2x_4 + 2x_5 + x_6 \geq 450$ → Width 5 Requirement

$x_2 + x_5 + 2x_6 \geq 120$ → Width 7 Requirement

$2x_1 + x_2 + x_4 \geq 200$ → Width 9 Requirement

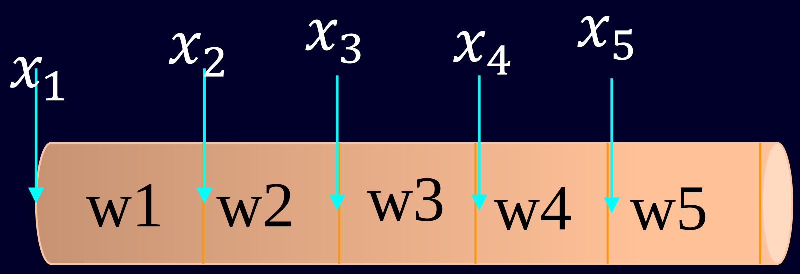
With all Variable Non-negative and integer

Example 10: Man Power Model

A Company, engaged in producing tinned food, has 300 trained employees on the rolls, *each of whom can produce one can of food in a week*, due to developing taste of the public for this kind of food, the *company plans to add to the existing labour force by employing 150 persons*, in a phased manner, over the next 5 weeks. The newcomers would have to undergo a *two-weeks training* program before being put to work. The training is to be given by employees from among the existing ones and it is known that *one employee can train three trainees*. Assume the training is off-the-job. However, the *trainees would be remunerated at the rate of $300 per week*, the same rate as for the trainers. The company has booked the following orders to supply during the next 5 weeks.

Formulate only the L.P. model to develop a training schedule that minimizes the labour cost over the five weeks period.

week	1	2	3	4	5
No. of cans	280	298	305	360	400



Employee starting at week 1 would get salary for all the five weeks

- Let x_i the Number of employee starting training at week i
- Minimize: Z=300[5 x_1+ 4 x_2 + 3 x_3 + 2 x_4 + x_5]
- Subject to: $x_1 + x_2 + x_3 + x_4 + x_5 = 150$
- Week1: $300 - \frac{x_1}{3} \geq 280$
- Week2: $300 - \frac{x_1}{3} - \frac{x_2}{3} \geq 298$
- Week3: $300 + x_1 - \frac{x_2}{3} - \frac{x_3}{3} \geq 305$
- *Week4*: $300 + x_1 + x_2 - \frac{x_3}{3} - \frac{x_4}{3} \geq 360$
- *Week5*: $300 + x_1 + x_2 + x_3 - \frac{x_4}{3} - \frac{x_5}{3} \geq 400$
- With all variables non-negative and integer

There are 300 trained employees available. Out of $\frac{x_1}{3}$ will be required to train x_1 trainees

Example 11: Product Mix Model

- A firm manufactures two items. It purchases castings which are then machined, bored and polished. Castings for item A and B costs *$2* and *$3* respectively and are sold at *$5* and *$6* respectively. Running costs of the three machines are *$20*, *$14* and *$17* per hour respectively. Capacities of the machines are as follows:

	Part A	Part B	Cost/hour
Machining capacity	25/hr	40/hr	20
Boring capacity	28/hr	35/hr	14
Polishing capacity	35/hr	25/hr	17

Formulate only the L.P. model to determine the product mix that maximizes the profit.

Product Mix Model

Let x_1 and x_2 the number of units of A and B to be manufactured per hour

	Part A	Part B	Cost/hour
Machining capacity	25/hr	40/hr	20
Boring capacity	28/hr	35/hr	14
Polishing capacity	35/hr	25/hr	17

Maximize: $Z = 1.2\, x_1 + 1.4 x_2$

Constraints are on the capacities of the machines. For ***<u>one hour</u>*** running of each machine

	Part A($)	**Part B($)**
Machining cost	20/25 =0.8	20/40=0.5
Boring cost	14/28=0.5	14/35=0.4
Polishing cost	17/35	17/25=0.7
Casting cost	2	3
Total cost	**3.8**	**4.6**
Selling Price	5	6
Profit	1.2	1.4

$$\frac{1}{25} x_1 + \frac{1}{40} x_2 \le 1$$

$$\frac{1}{28} x_1 + \frac{1}{35} x_2 \le 1$$

$$\frac{1}{35} x_1 + \frac{1}{25} x_2 \le 1$$

$$x_1 \, and \; x_2 \ge 0$$

Example 12: Investment Model

- A bank is in the process of formulating its loan policy involving a maximum of ***$600 million***. Table below gives the relevant types of loans. Bad debts are not recoverable and produce no interest revenue. To meet competition from other banks, the following policy guidelines have been set: *at least 40% of the funds must be allocated to the agricultural and commercial loans*. *Funds allocated to the houses must be at least 50% of all loans given to personal, car*. The *overall bad debts on all loans may not exceed 0.06*. Formulate the Linear Program Model to determine the optimal loan allocation.

Type of Loan	Interest rate (%)	Bad debt (probability)
Persnal	17	0.10
Car	14	0.07
Housing	11	0.05
Agriculture	10	0.08
Commercial	13	0.06

Investment Model

- *Decision Variables:*
- Let $x_1: the\ amont\ of\ money\ allocated\ to\ personal\ Loan$
- Let $x_2: the\ amont\ of\ money\ allocated\ to\ Car\ Loan$
- Let $x_3: the\ amont\ of\ money\ allocated\ to\ House\ Loan$
- Let $x_4: the\ amont\ of\ money\ allocated\ to\ Farming\ Loan$
- Let $x_5: the\ amont\ of\ money\ allocated\ to\ commercial\ Loan$

Type of Loan	Interest rate (%)	Bad debt (probability)	Good standing Loan
Persnal	17	0.10	0.90
Car	14	0.07	0.93
Housing	11	0.05	0.95
Agriculture	10	0.08	0.92
Commercial	13	0.06	0.94

- Objective is to Maximize the net return
- $net\ return = total\ interest - lost\ bad\ debt$
- $total\ interest$=$0.17(0.90x_1) + 0.14(0.93x_2) + 0.11(0.95x_3) + 0.1(0.92x_4) + 0.13(0.94x_5)$
- Lost Bad debt= $0.1x_1 + 0.07x_2 + 0.05x_3 + 0.08x_4 + 0.06x_5$

Example 12: Investment Model

- **Subject to**: $x_1 + x_2 + x_3 + x_4 + x_5 \leq 600$
- $x_4 + x_5 \geq 0.4(x_1 + x_2 + x_3 + x_4 + x_5)$
- $x_3 \geq 0.5(x_1 + x_2)$
- $0.1x_1 + 0.07x_2 + 0.05x_3 + 0.08x_4 + 0.06x_5 \leq 0.06\ (x_1 + x_2 + x_3 + x_4 + x_5)$
- With all variables nonnegative

Example 13: Machining Model

- A factory manufactures a product *each unit* of which consists of *5 units of part A and 4 units of part B*. the two parts A and B require different *raw materials of which 120 units and 240* units respectively are available. These parts can be manufactured by three different methods. Raw material requirements per production run and the number of units for each part produced are given below.

Determine the number of production runs for each method so as to maximize the total number of complete units of the final product.

Method	Input per run (units)		Output per run (units)	
	Raw material I	Raw material II	Part A	Part B
1	7	5	6	4
2	4	7	5	8
3	2	9	7	3

Example 13

Method	Input per run (units)		Output per run (units)	
	Raw material I	Raw material II	Part A	Part B
1	7	5	6	4
2	4	7	5	8
3	2	9	7	3

- Let x_1: the number of runs of Method1
- Let x_2: the number of runs of Method2
- Let x_3: the number of runs of Method3
- The Number of Part A unit to be produced $6\ x_1 + 5\ x_2 + 7x_3$
- The Number of Part B unit to be produced $4\ x_1 + 8\ x_2 + 3x_3$
- The Number of complete units to be produced

$$Y = \mathrm{Min}\left(\frac{6\ x_1 + 5\ x_2 + 7x_3}{5}, \frac{4\ x_1 + 8\ x_2 + 3x_3}{4}\right)$$

Method	Input per run (units)		Output per run (units)	
	Raw material I	Raw material II	Part A	Part B
1	7	5	6	4
2	4	7	5	8
3	2	9	7	3

- Maximize Z=Y
- Subject to:
- $7\, x_1 + 4\, x_2 + 2x_3 \leq 120$
- $5\, x_1 + 7\, x_2 + 9x_3 \leq 240$
- With all variables Non-negative and integer


Thank You