

Sets of numbers :

• $C : \{1, 2, 3, \dots, \infty\}$

• $N : \{0, 1, 2, \dots, \infty\}$

• $Z : \{\dots, -2, -1, 0, 1, 2, \dots, \infty\}$

• Q : Rational numbers

• Q' : Irrational numbers $e, \pi, \sqrt{2}$

• $R : Q \cup Q' = \{ \text{Note } Q \cap Q' = \emptyset \}$

New set of numbers : Complex numbers

$$C = \{x + yi, x, y \in R\}$$

o Conjugate

Ex. if $a = 2 + 3i$

Then conj of $a = 2 - 3i$

Note
Care

• The sum of a number and its conjugate is a Real number

• when subtracting the number from its conj we get Pure imaginary number

• when Multiplying the number and its Conjugate we get a Real number

@ Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & \dots \\ \dots & \dots & \dots & \dots \\ a_{n1} & \dots & \dots & a_{nm} \end{bmatrix} (n \times m)$$

- size of Matrix $(n \times m) \equiv$ (num of Rows \times num of cols)
- Matrices are identical when
 - they are the same size
 - every element in the first equals every element in the 2nd Matrix

Addition and subtraction

Constraints: They must be the same size

$$\text{Ex. } A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} + & -1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad A-B = \begin{bmatrix} 0 & 1 & -1 \\ 3 & -3 & -1 \end{bmatrix}$$

* addition properties

- associative $(A+B)+C = A+(B+C)$
- Comutitive $A+B = B+A$
- identity $A+O = O+A = A$
- inverse $A+(-A) = O$

Scalar Multiplication

Multiplying a Constant with a Matrix

$$\text{Ex. } A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & 0 \end{bmatrix}$$

$$k \times A = \begin{bmatrix} 1k & 2k & 0k \\ 3k & 2k & 0k \end{bmatrix}$$

$$k(A+B) = kA + kB$$

$$A(k+h) = kA + hA$$

Transpose

Making rows columns and Making columns Rows

$$\text{Ex. } A^T = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 0 & 0 \end{bmatrix}$$

$$A^T + B^T = (A+B)^T$$

if $A = A^T$

- then A is squared Matrix
- A is symmetric

if $A = -A^T$

- A is squared
- diagonal elements equal to 0
- skew-symmetric

$$A + A^T$$

$$\therefore (A + A^T)^T$$

$$= A^T + A$$

$\therefore A + A^T$ is symmetric

$$A - A^T$$

$$\therefore (A - A^T)^T$$

$$= A^T - A$$

$\therefore A - A^T$ is skew symmetric

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

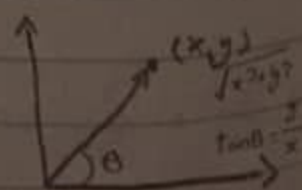
adjoint Matrix C^*

How to make C adjoint

• First we Transpose C

• Generate the conjugate of the result

inner product / scalar product



$$x \cdot y = [x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots + x_n y_n]$$

$$= x^T y$$

$$\begin{matrix} x & y \\ \left[\begin{matrix} \\ \\ \\ \end{matrix} \right]_{n \times 1} & \left[\begin{matrix} \\ \\ \\ \end{matrix} \right]_{n \times 1} \end{matrix}$$

names $\|x\| = \sqrt{x \cdot x} = \sqrt{x \cdot x}$

Trace of Matrix = Sum of Diagonal elements