OPERATIONAL RESEARCH

Section1

Section 1

- We will learn to solve "Optimization Problems" and Find "Optimal Solution"

واحد عنده مشكلة وعنده Objective لازم يحققه وعايز يوصل
للـ Optimal Sol.

- Each problem has two essential parts

1. Objective: → Maximize "profit, production, ..."
   → Minimize "Time, Cost, Effort, Risk, ..."

2. Subject to: Constraints

قيود لبعض أصح، مثلاً لازم ينتج كمية معينة، يكسب
فلوس معينة، يشتغل عدد أيام معينة

Problems

Linear	Non-Linear
"Power 1"	"Sin, Cos, e, ..."
$\hookrightarrow 2x_1 + 4x_2 \le 5$	

لازم الـ Obj والـ Cons. يبقوا lin

3- Problem Formulation:

→ Optimization problems must often are stated Verbally.

The Solution procedure is to model the problem with a mathematical program and solve the program by using several techniques.

Step 1: Determine the quantity to be obtimized and express it as a mathematical function.

Step 2: Identify all requirements, restrications and limitations and express it as a mathematical function.

Step 3: Express any hidden Conditions. Generally they involve non-negativity or integer requirements on the input variables.

John must work at least 20 hours a week to supplement his income while attending school. He has the opportunity to work in two retail stores. In store 1, he can work between 5 and 12 hours a week, and in store 2 he is allowed between 6 and 10 hours. Both stores pay the same hourly wage. In deciding how many hours to work in each store, John wants to base his decision on work stress. Based on interviews with present employees, John estimates that, on an ascending scale of 1 to 10, the stress factors are 8 and 6 at stores 1 and 2, respectively. Because stress mounts by the hour, he assumes that the total stress for each store at the end of the week is proportional to the number of hours he works in the store. How many hours should John work in each store?

Problem 1: "In Sheet One"

- John must work at least 20 hours a week.
- In store 1, he can work between 5, 12 hrs/week.
- In store 2, he can work between 6, 10 hrs/week.
- To base his decision on "work stress"
- Stress factor of store 1 = 8/hour
- Stress factor of store 2 = 6/hour.
- Total stress for each store is proportional to the total hrs/week.

How many hours should he work in each store?!

1. Decision variables:

$x_1 \rightarrow$ No. of working hours in store 1 per week.

$x_2 \rightarrow$ No. of working hours in store 2 per week.

2. Objective:

Minimize stress $Z = 8x_1 + 6x_2$

3. Subject to:

$x_1 + x_2 \geq 20$

$x_1 \geq 5$ and $x_1 \leq 12$

$x_2 \geq 6$ and $x_2 \leq 10$

hidden const.: $x_1, x_2 \geq 0$

A Blending Problem: A manufacturer of artificial sweetener blends 14 kilograms of saccharin and 18 kilograms of dextrose to prepare two new products: Sweet and Low-Sugar. Each kilogram of Sweet contains 0.4 kilograms of dextrose and 0.2 kilograms of saccharin, while each kilogram of Low-Sugar contains 0.3 kilograms of dextrose and 0.4 kilograms of saccharin. If the profit on each kilogram of Sweet is 20 cents and the profit on each kilogram of Low-Sugar is 30 cents, how many kilograms of each product should be made to maximize the profit?

Problem 15 : Blendling Problem

Saccharin "S" 14 kg

Dextrose "D" 18 kg

Blends

Sweet Contains
→ 0.4 D
→ 0.2 S
profit 20 Cent

low Sugar Contains
→ 0.3 D
→ 0.4 S
profit 30 Cent

- How many kg should be made to maximize the profit ?!

1 Decision Variables:

x_1 → No. of kgs of Sweet

x_2 → No. of kgs of low Sugar.

2 objective:

Maximize profit : $Z = 20x_1 + 30x_2$

3 Subject to:

$0.4x_1 + 0.3x_2 \le 18$

$0.2x_1 + 0.4x_2 \le 14$

hidden Constraints:

$x_1, x_2 \ge 0$

Show & Sell can advertise its products on local radio and television (TV). The advertising budget is limited to $10,000 a month. Each minute of radio advertising costs $15 and each minute of TV commercials $300. Show & Sell likes to advertise on radio at least twice as much as on TV. In the meantime, it is not practical to use more than 400 minutes of radio advertising a month. From past experience, advertising on TV is estimated to be 25 times as effective as on radio. Determine the optimum allocation of the budget to radio and TV advertising.

Problem 4:

- Total budget is limited to 10,000$/month
- 1 min on Radio Cost 15$
  1 min on TV Cost 300$
- They like to advertise on radio at least twice on the TV.
- It's not practical to advertise more than 400 min/month on radio
- Advertising on TV is estimated to be 25 as effective as on Radio.

- Determine the optimum allocation of the budget to Radio and TV advertising?! (effectiveness)

1 Decision Variables:

$X_1 \rightarrow$ # of minutes on radio/month

$X_2 \rightarrow$ # of minutes on TV/month

2 objective:

Maximize effectiveness: $Z = X_1 + 25X_2$

3 Subject to:

$15X_1 + 300X_2 \leq 10{,}000$

$X_1 \leq 400$

$X_1 \geq 2X_2$

hidden: $X_1, X_2 \geq 0$

2. After modeling the problem, there is more than one method to solve by:

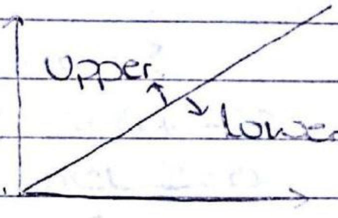
Graphically (2 variables) | Theoritically

Note to Keep in mind:

→ Each line split the space into "3 parts" "Upper, Lower, Line itself"

Upper → Most of times > f_n
lower → Most of times < f_n
Line → Always = f_n

To know and Be accurate, Substitute with any point



Example 1:

Min $z = 8x_1 + 6x_2$

Subj. to : $x_1 + x_2 \geq 20$

$x_1 \geq 5$ and $x_2 \leq 12$

$x_2 \geq 6$ and $x_2 \leq 10$

$x_1, x_2 \geq 0$

$x_1 + x_2 = 20 \rightarrow x_1 = 0 \quad x_2 = 20 \quad \rightarrow ①$

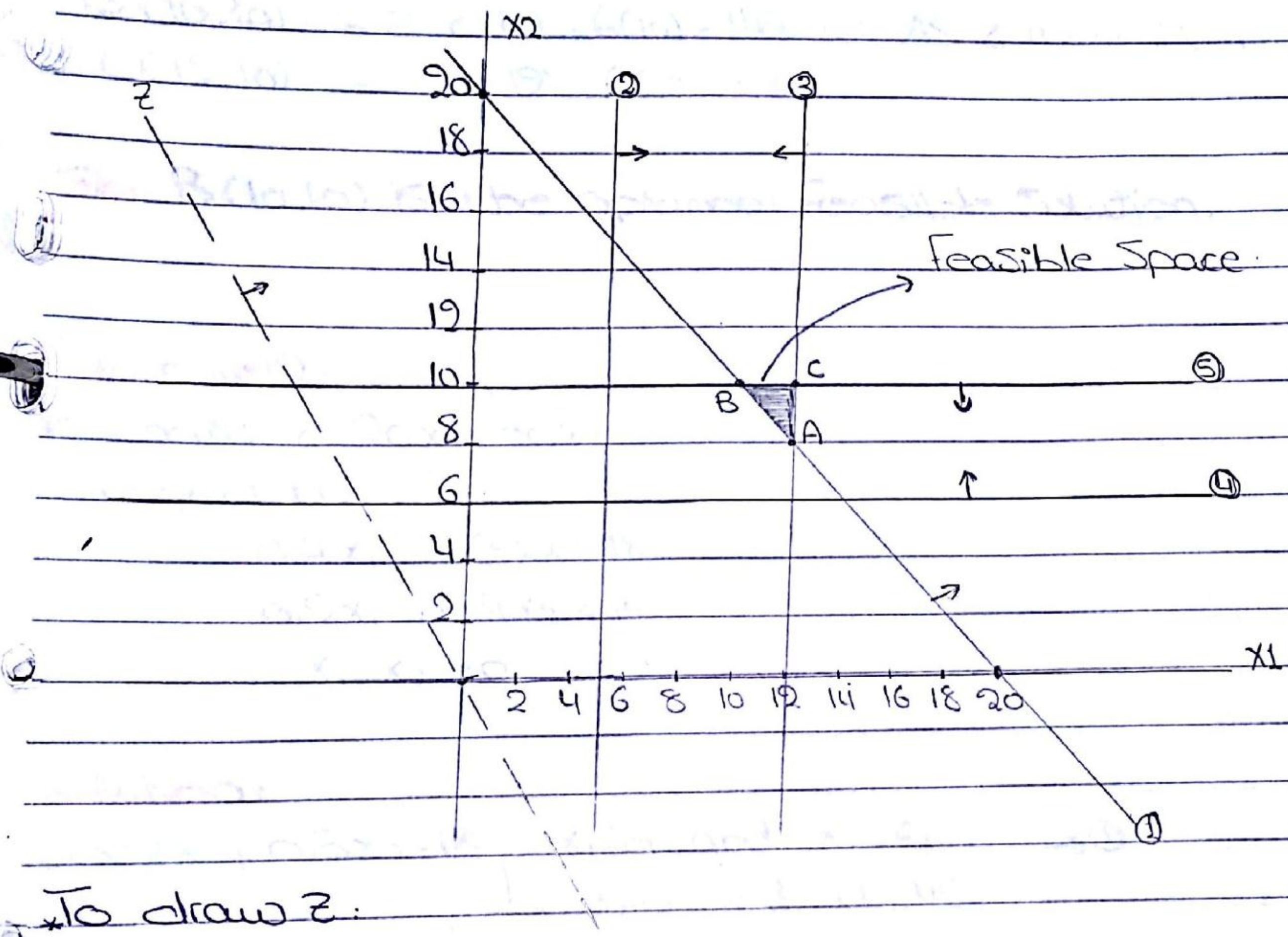
$\quad\quad\quad\quad\quad\quad \rightarrow x_2 = 0 \quad x_1 = 20$

$x_1 = 5 \rightarrow$ ② $\quad x_2 = 0 \rightarrow x_1 = 5$

$x_1 = 12 \rightarrow$ ③ $\quad x_2 = 0 \rightarrow x_1 = 12$

$x_2 = 6 \rightarrow$ ④ $\quad x_1 = 0 \rightarrow x_2 = 6$

$x_2 = 10 \rightarrow$ ⑤ $\quad x_1 = 0 \rightarrow x_2 = 10$



* To draw Z:
  - $\rightarrow Z = ax_1 + bx_2$
    1. let $Z = 0$
    2. $(0,0), (b,-a), (-b,a)$

So, Apply on the above problem.

$Z = 8x_1 + 6x_2$

$\rightarrow (0,0), (6,-8), (-6,8)$

* دايما الـ Solution هيكون واحد من الـ Corners
* أو انني ارسم الـ Z واشوف الـ Feasible Sol.

* Z بتزيد لفوق ، افضل ماشي لحد أول نقطة اخبط فيها من الـ Feasible Space

- We will Solve using "Substitution", Each point resulting from the intersection of 2 Lines

A (12,8) $\rightarrow$ $Z = 8(12) + 6(8) = 144$

B (10,10) $\rightarrow$ $Z = 8(10) + 6(10) = 140$ ✓ As Z minimizes.

C (12,10) $\rightarrow$ $Z = 8(12) + 6(10) = 156$

So, B(10,10) is the optimal Feasible Solution.

Example 2:

Maximize $Z = 20x_1 + 30x_2$

Subject to:

$$0.4x_1 + 0.3x_2 \le 18$$

$$0.2x_1 + 0.4x_2 \le 14$$

$$x_1, x_2 \ge 0$$

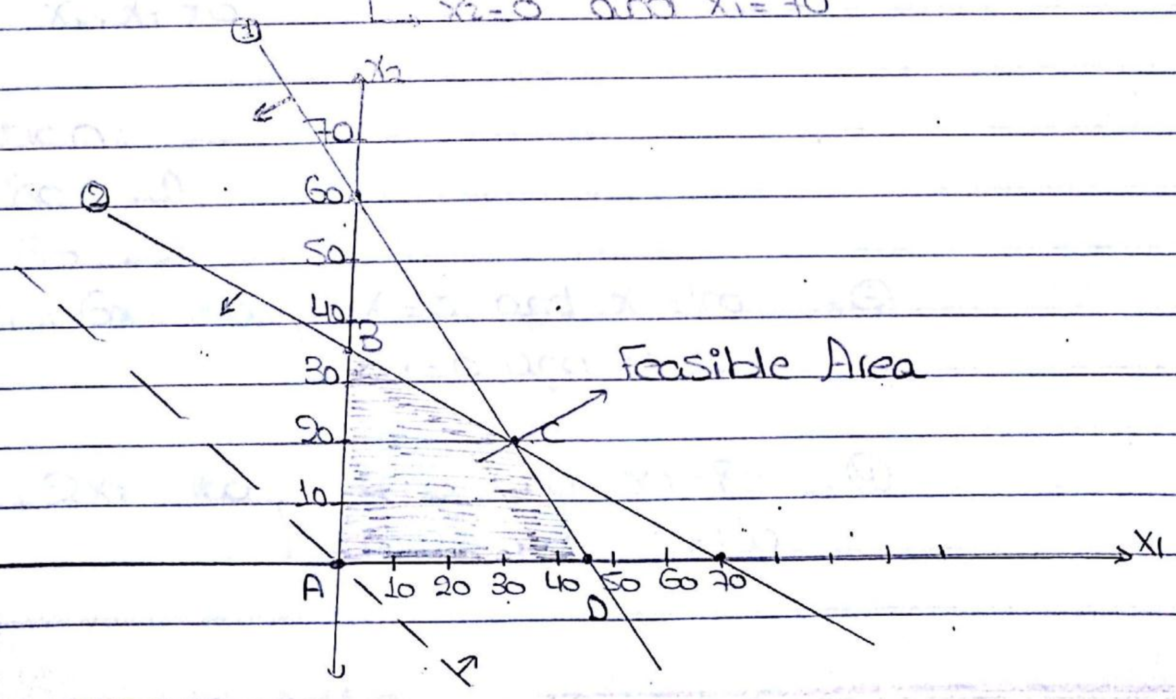
Solution:

* $0.4x_1 + 0.3x_2 = 18$ $\rightarrow$ $x_1 = 0$ and $x_2 = 60$ $\rightarrow$ ①

$\rightarrow$ $x_2 = 0$ and $x_1 = 45$

* $0.2x_1 + 0.4x_2 = 14$ $\rightarrow$ $x_1 = 0$ and $x_2 = 35$ $\rightarrow$ ②

$\rightarrow$ $x_2 = 0$ and $x_1 = 70$



* To draw Z line

$Z = 20x_1 + 30x_2$

$\rightarrow (0,0), (-30,20), (30,-20)$

وبما ان ال Z بـ Maximize ف يبقى لو مشينا بال Z line هنحتاج نختار آخر point في ال Feasible Area

* Corners of the Feasible area:

$A(0,0) \Rightarrow Z = 20(0) + 30(0) = 0$

$B(0,35) \rightarrow Z = 20(0) + 30(35) = 1050$

$C(30,20) \rightarrow Z = 20(30) + 30(20) = 1200$ ✓ As Z maximizes

$D(45,0) \rightarrow Z = 20(45) + 30(0) = 900$

So, $x_1 = 30$ and $x_2 = 20$ is the "Optimal Feasible Sol."

Example 3:

* Maximize: $Z = x_1 + 2x_2$
* Subj. to:

$x_1 \leq 80$ and $x_2 \leq 60$

$5x_1 + 6x_2 \leq 600$

$x_1 + 2x_2 \leq 160$

$x_1, x_2 \geq 0$

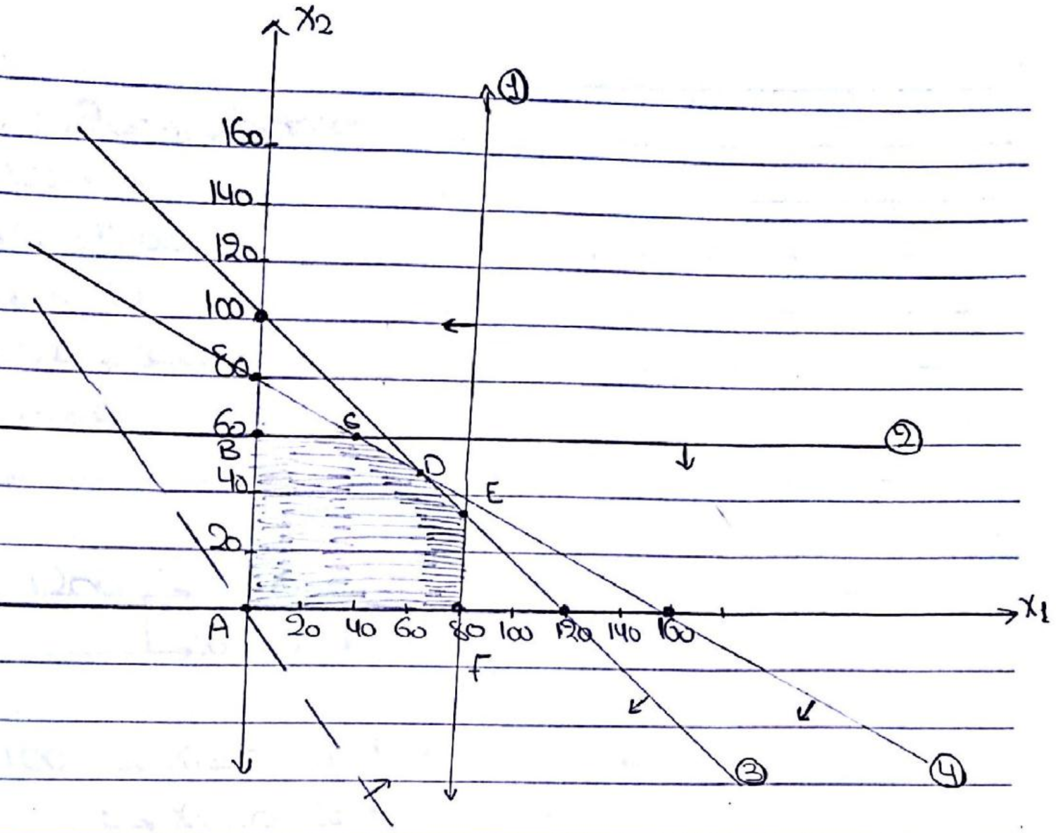
Solution:

* $x_1 = 80 \rightarrow (1)$
* $x_2 = 60 \rightarrow (2)$
* $5x_1 + 6x_2 = 600 \rightarrow x_1 = 0$ and $x_2 = 100 \rightarrow (3)$

$\quad \hookrightarrow x_2 = 0$ and $x_1 = 120$

* $x_1 + 2x_2 \leq 160 \rightarrow x_1 = 0$ and $x_2 = 80 \rightarrow (4)$

$\quad \hookrightarrow x_2 = 0$ and $x_1 = 160$



* To draw Z-line:

→ (0,0), (-20,10), (20,-10) "Increase"

* Corners:

A(0,0) → Z = 1(0) + 2(0) = 0

B(0,60) → Z = (0) + 2(60) = 120

C(40,60) → Z = 40 + 2(60) = 160

D(60,50) → Z = 60 + 2(50) = 160 → 2 optimal Solutions

E(80,33.3) → Z = 80 + 2(33.3) = 146.6

F(80,0) → Z = 80 + 2(0) = 80

Since, there is more than one optimal Sol.
Therefore, this problem has "Infinite no. of Sol."

Example 4:

* Maximize: $Z = 200x_1 + 300x_2$
* Subject to:

$2x_1 + 3x_2 \geq 1200$

$x_1 + x_2 \leq 400$

$2x_1 + 1.5x_2 \geq 900$

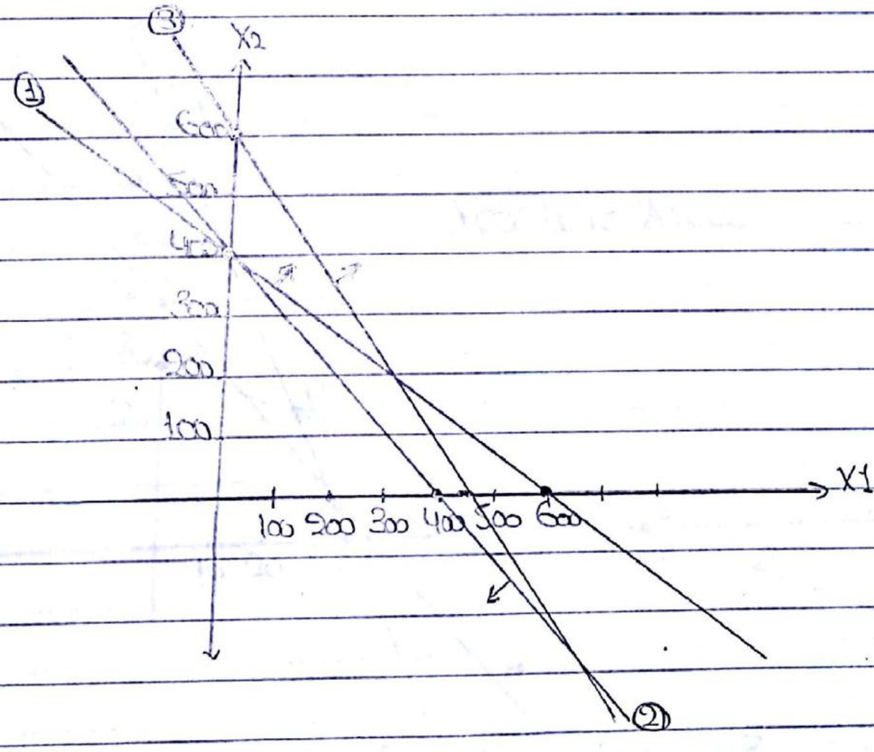
$x_1, x_2 \geq 0$

Solution:

* $2x_1 + 3x_2 = 1200 \rightarrow x_1 = 0$ and $x_2 = 400 \rightarrow$ ①
  $\quad \rightarrow x_2 = 0$ and $x_1 = 600$

* $x_1 + x_2 = 400 \rightarrow x_1 = 0$ and $x_2 = 400 \rightarrow$ ②
  $\quad \rightarrow x_2 = 0$ and $x_1 = 400$

* $2x_1 + 1.5x_2 = 900 \rightarrow x_1 = 0$ and $x_2 = 600 \rightarrow$ ③
  $\quad \rightarrow x_2 = 0$ and $x_1 = 450$



No Feasible Area., No Feasible Solution.

Example 5:

* Maximize: $Z = 400x_1 + 600x_2$
* Subject to:

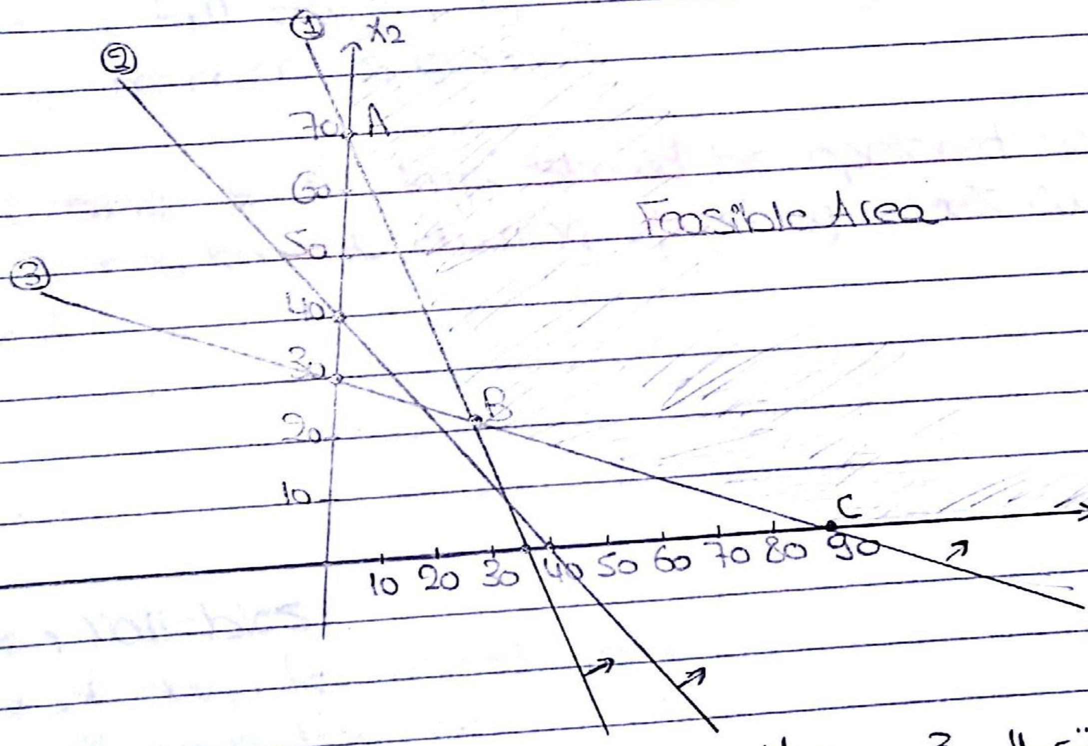
$$2x_1 + x_2 \geq 70$$
$$x_1 + x_2 \geq 40$$
$$x_1 + 3x_2 \geq 90$$
$$x_1, x_2 \geq 0$$

Solution:

* $2x_1 + x_2 = 70 \rightarrow x_1 = 0$ and $x_2 = 70 \rightarrow$ ①
  $\hookrightarrow x_2 = 0$ and $x_1 = 35$

* $x_1 + x_2 = 40 \rightarrow x_1 = 0$ and $x_2 = 40 \rightarrow$ ②
  $\hookrightarrow x_2 = 0$ and $x_1 = 40$

* $x_1 + 3x_2 = 90 \rightarrow x_1 = 0$ and $x_2 = 30 \rightarrow$ ③
  $\hookrightarrow x_2 = 0$ and $x_1 = 90$



*دلوقت الـ Z بـ Max ، و الـ Feasible Area بـ unbounded ، يبقى كدا
ما هقفش بالـ Z كل ما هلاقي Solution أحسن
يبقى عمري ما هوصل للـ Optimal Sol.

"Non-bounded Sol."